

**SILESIAAN UNIVERSITY OF TECHNOLOGY**  
**FACULTY OF ENERGY AND ENVIRONMENTAL ENGINEERING**

# **TESTS ON THE TRANSFORMER**

## **(E-13)**

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# TESTS ON THE TRANSFORMER

## 1. Aim of exercise

The aim of this exercise is to determine the transformer no-load and short-circuit characteristics and to analyze properties of the transformer as an electric machine. The measurements will make it possible to find the values of parameters of the transformer equivalent circuit elements.

## 2. Introduction

The transformer is a static electric machine used to transform electrical energy. The aim of the transformation is to raise or lower the voltage, which results in lower or higher values of the current intensity. The energy transformation takes place via a magnetic field. A simplified diagram of the transformer structure is presented in Fig. 1.1.

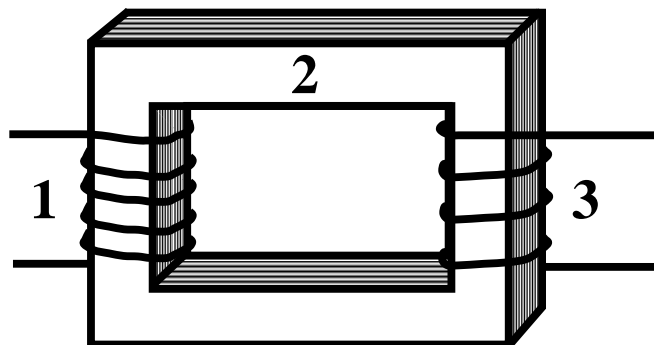


Fig. 1.1. Simplified diagram of the transformer structure

Windings {1} and {3} are wound on ferromagnetic core {2}. The windings are insulated from the core. The transformer core, constituting the magnetic circuit of the transformer, is usually made of thin metal sheets which are insulated from each other. The windings are made of insulated copper wire. Like for any electric machine, the transformer properties are described for three typical states: operation under no-load, on-load and short-circuit conditions.

During the transformer operation, losses of active power occur in the core due to eddy currents and magnetic hysteresis and in the windings because of the copper loss – electric power dissipated due to resistance of the windings.

## 2.1. Transformer no-load state

The transformer is in the no-load state if its primary winding is supplied with complex supply  $\underline{U}_1$  from the source, and the secondary circuit is under no-load current conditions (secondary side current  $\underline{I}_2 = 0$ ). The value of no-load current  $\underline{I}_0$  is from several to about fifteen per cent of the value of the primary side rated current. Flowing through the primary winding, in the transformer core current  $\underline{I}_0$  produces main magnetic flux  $\underline{\Phi}$  and primary side magnetic leakage flux  $\underline{\Phi}_{1R}$  closing through air (there is no current flowing through the secondary winding and therefore no secondary side magnetic flux leakage  $\underline{\Phi}_{2R}$  is produced). The main flux induces respective electromotive forces in the windings:  $\underline{E}_1$  and  $\underline{E}_2 = \underline{U}_2$ . Primary side magnetic leakage flux  $\underline{\Phi}_{1R}$  induces electromotive force  $\underline{E}_{1R} = \underline{U}_{X1}$ . A diagram of the transformer that takes the generated magnetic fluxes into account is presented in Fig. 1.2.

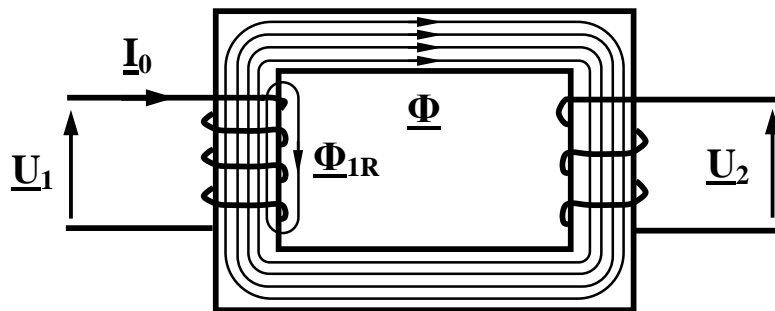


Fig. 1.2. Magnetic fluxes in the transformer core under no-load conditions

The analysis of the operation of the transformer (or other electric machines) may be performed conveniently based on the equivalent circuit diagram and on the vector diagram of voltages and currents. An equivalent circuit diagram is complete if it takes account of all essential phenomena occurring during a given machine operation. Making an equivalent circuit diagram of a transformer under no-load conditions, the following elements need to be taken into consideration:

- $X_{\mu}$  – magnetizing reactance related to main magnetic flux  $\underline{\Phi}$ ,
- $X_{1R}$  – magnetizing reactance related to primary side magnetic leakage flux  $\underline{\Phi}_{1R}$ ,
- $R_1$  – winding resistance of the transformer primary side,
- $R_{Fe}$  – equivalent core-loss resistance (resulting from hysteresis and eddy currents).

Power losses in insulating materials, capacity currents and leakage currents are usually ignored in the making of the equivalent circuit diagram. The transformer secondary side is not shown in the diagram, either (secondary side current  $\underline{I}_2 = 0$ ). A transformer in the no-load state behaves like an element with a steel core (reactor).

The equivalent circuit diagram and the vector diagram of a transformer under no-load conditions are presented in Fig. 1.3.

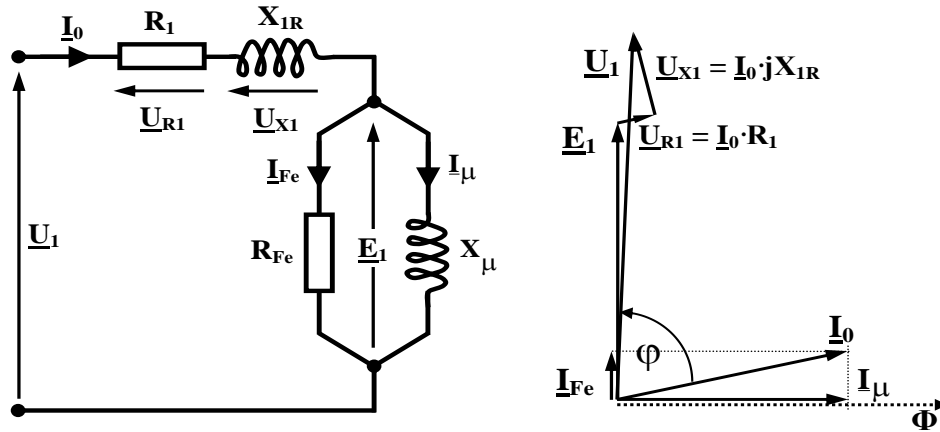


Fig. 1.3. Equivalent circuit diagram and vector diagram of a transformer under no-load conditions

Kirchoff's equation may be written for the diagram presented in Fig. 1.3:

$$\underline{U}_1 = \underline{U}_{R1} + \underline{U}_{X1} + \underline{E}_1 = R_1 \cdot \underline{I}_0 + jX_{1R} \cdot \underline{I}_0 + jX_{\mu} \cdot \underline{I}_{\mu} \quad (1)$$

All the active power consumed by a transformer under no-load conditions is converted into heat [4] and these losses are almost exclusively core losses. Knowing the transformer primary winding resistance  $R_1$  and the power consumed in the no-load state  $P_0$ , the following may be written:

$$P_0 - I_0^2 \cdot R_1 = \Delta P_{Fe} = \Delta P_H + \Delta P_W \quad (2)$$

where:

$$\begin{aligned} \Delta P_{Fe} = \Delta P_H + \Delta P_W & \quad - \quad \text{core loss,} \\ \Delta P_H & \quad - \quad \text{hysteresis loss,} \\ \Delta P_W & \quad - \quad \text{eddy-current loss.} \end{aligned}$$

Knowing the value of the total copper loss  $\Delta P_{Fe}$  depending on frequency  $f$ , the shares of the hysteresis loss and the eddy-current loss may be determined from:

$$\Delta P_H = k_H \cdot B_m^2 \cdot f \quad (3)$$

$$\Delta P_W = k_W \cdot B_m^2 \cdot f^2 \quad (4)$$

where:

$B_m$  – amplitude of magnetic induction,  
 $f$  – frequency of supply voltage (independent variable),  
 $k_H, k_W$  – constant coefficients which may be determined from regression analysis (5),

$$\frac{P_0 - I_0^2 \cdot R_1}{f \cdot B_m^2} = k_H + k_W \cdot f. \quad (5)$$

For constant supply frequency  $f$ , by measuring the values of power  $P_0$ , current  $I_0$ , primary side voltage  $U_1$  and secondary side voltage  $U_2$ , and knowing resistance  $R_1$ , it is possible to find:

- no-load state power factor  $\quad - \cos\varphi_0 = \frac{P_0}{U_1 \cdot I_0} \quad , \quad (6)$

- transformer ratio  $\quad - \vartheta = \frac{U_1}{U_2} \quad , \quad (7)$

- core-loss current  $\quad - I_{Fe} = \frac{P_0 - I_0^2 \cdot R_1}{E_1} \approx \frac{P_0 - I_0^2 \cdot R_1}{U_1} \quad , \quad (8)$

- magnetizing current  $\quad - I_\mu = \sqrt{I_0^2 - I_{Fe}^2} \quad , \quad (9)$

and the equivalent circuit diagram approximate parameters  $R_{Fe}$  and  $X_\mu$  :

$$X_\mu = \frac{E_1}{I_\mu} \cong \frac{U_1}{I_\mu} \approx \frac{U_1}{I_0} \quad , \quad (10)$$

$$R_{Fe} = \frac{E_1^2}{\Delta P_{Fe}} \cong \frac{U_1^2}{P_0 - I_0^2 \cdot R_1} \approx \frac{U_1^2}{P_0} \quad . \quad (11)$$

## 2.2. Transformer on-load state

The transformer is in the on-load state if its primary winding is supplied with complex supply voltage  $\underline{U}_1$  from the source, and an element with impedance  $\underline{Z}$  is connected to the secondary circuit. Currents  $\underline{I}_1$  and  $\underline{I}_2$  flowing through the primary and secondary winding generate in the transformer core main magnetic flux  $\underline{\Phi}$ , primary side magnetic leakage flux  $\underline{\Phi}_{1R}$  and secondary side magnetic leakage flux  $\underline{\Phi}_{2R}$ , which are closing through air (flux  $\underline{\Phi}_{2R}$  induces electromotive force (EMF)  $\underline{E}_{2R} = \underline{U}_{X2}$ ). A diagram of the transformer that takes the generated magnetic fluxes into account is presented in Fig. 1.4.

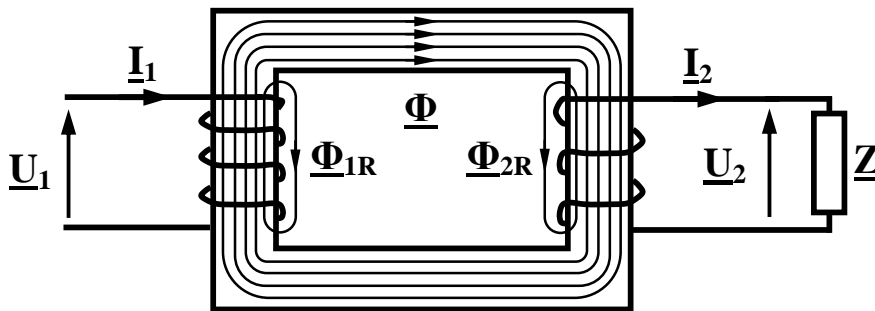


Fig. 1.4. Magnetic fluxes in the transformer core under on-load conditions

Secondary side current  $\underline{I}_2$  depends on voltage  $\underline{U}_2$  at the secondary winding terminals and on the parameters of element  $\underline{Z}$ . Primary side current  $\underline{I}_1$  adapts to secondary side current  $\underline{I}_2$  and to no-load current  $\underline{I}_0$ . No-load current  $\underline{I}_0$  has two components: magnetizing current  $\underline{I}_\mu$ , necessary to magnetize the core circuit, and core-loss current  $\underline{I}_{Fe}$  representing the total loss in the core. Making an equivalent circuit diagram of a transformer under on-load conditions (apart from the elements mentioned in 2.1 above), the following quantities need to be taken into consideration:

- $X_{2R}$  – inductive reactance related to the secondary side magnetic leakage flux  $\underline{\Phi}_{2R}$ ,
- $R_2$  – winding resistance of the transformer secondary side.

The equivalent circuit diagram of a transformer under on-load conditions is presented in Fig. 1.5.

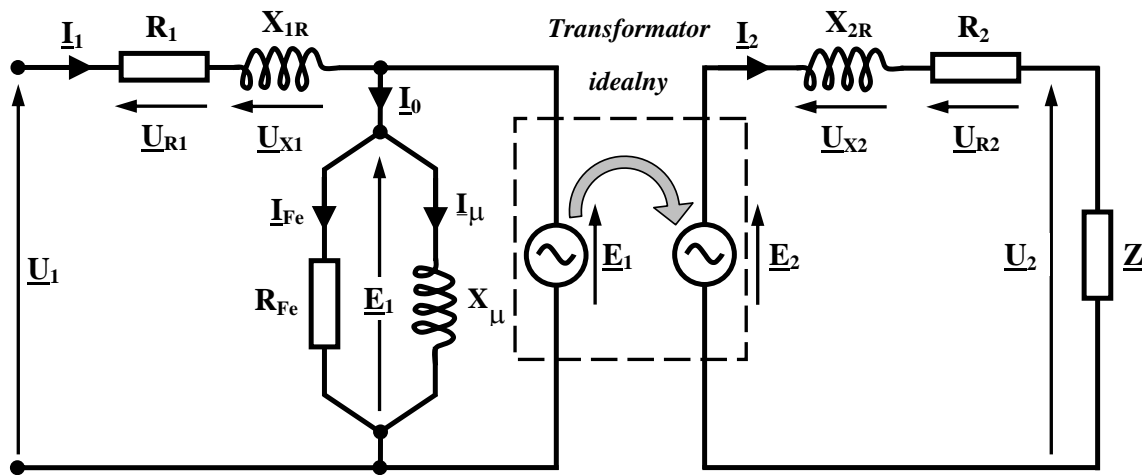


Fig. 1.5. Equivalent circuit diagram of a transformer under on-load conditions (form I)

do rysunku: Ideal transformer

For the diagram presented in Fig. 1.5, Kirchoff's equations may be written both for the primary side circuit, where it takes the form of equation (1):

$$\underline{U}_1 = \underline{U}_{R1} + \underline{U}_{X1} + \underline{E}_1, \quad (12)$$

and for the secondary side as:

$$\underline{E}_2 = \underline{U}_{X2} + \underline{U}_{R2} + \underline{U}_2. \quad (13)$$

The transformer equivalent circuit diagram under on-load (and short-circuit) conditions is very often presented without an ideal transformer, expressing the quantities of the secondary side in the primary side terms (in form II). The conversion of secondary side quantities into primary side quantities is carried out using the notion of the ideal transformer ratio or an equivalent notion – the transformer turns ratio  $\mathcal{G}_N$ .

$$\mathcal{G}_N = \frac{E_1}{E_2} = \frac{N_1}{N_2}, \quad (14)$$

where:

$N_1$  – number of turns of primary side winding,

$N_2$  – number of turns of secondary side winding,

The secondary side quantities converted (calculated) into primary side quantities are marked with the **prime symbol** ( $'$ ). Finally, the following is obtained:

- representation of voltage on the secondary side to the primary side

$$E'_2 = E_2 \cdot \mathcal{G}_N, \text{ because: } E'_2 = E_2 \cdot \frac{E_1}{E_2} = E_1, \text{ or generally } U'_2 = U_2 \cdot \mathcal{G}_N, \quad (15)$$

- representation of current on the secondary side to the primary side

$$I'_2 = I_2 \cdot \frac{1}{\mathcal{G}_N}, \quad (16)$$

- representation of the secondary side resistance to the primary side

$$R'_2 = R_2 \cdot \mathcal{G}_N^2, \quad (17)$$

$$\text{because: } R'_2 = \frac{U'_2}{I'_2} = \frac{U_2 \cdot \mathcal{G}_N}{I_2 \cdot \frac{1}{\mathcal{G}_N}} = R_2 \cdot \mathcal{G}_N^2,$$

- representation of the secondary side reactance to the primary side

$$X'_2 = X_2 \cdot \mathcal{G}_N^2, \quad (18)$$

- representation of the secondary side impedance to the primary side

$$\underline{Z}'_2 = R'_2 + jX'_2, \quad (19)$$

- representation of the secondary side apparent power to the primary side

$$S'_2 = S_2, \quad (20)$$

the power is **invariant** because:  $S'_2 = I'_2 \cdot U'_2 = I_2 \cdot U_2 = S_2$ ,

- representation of the secondary side phase difference to the primary side

$$\varphi'_2 = \varphi_2, \quad (21)$$

the phase difference is **invariant** because:

$$\varphi'_2 = \arctg \frac{X'_2}{R'_2} = \arctg \frac{X_2 \cdot \mathcal{G}_N^2}{R_2 \cdot \mathcal{G}_N^2} = \arctg \frac{X_2}{R_2} = \varphi_2.$$

Depending on the needs, quantities may be converted in either direction: from the secondary to the primary side or vice versa. The equivalent circuit diagram of a transformer under on-load conditions, after the secondary side has been converted into the primary side is presented in Fig. 1.6.

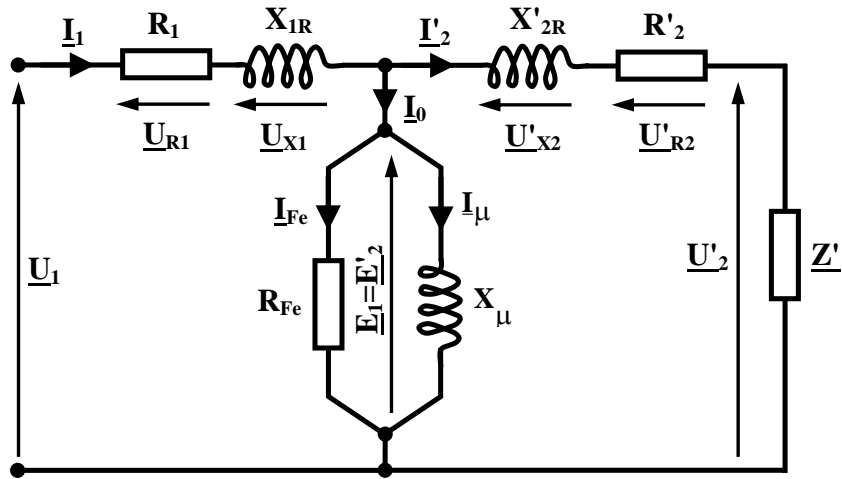


Fig. 1.6. Equivalent circuit diagram of a transformer under on-load conditions (form II)

The transformer equivalent diagram circuit in the form of connected electric circuits of the primary and secondary side makes it possible to make a vector diagram of currents and voltages and analyze the transformer operation under on-load conditions in a convenient way. The vector diagram of a transformer under on-load conditions is presented in Fig. 1.7.

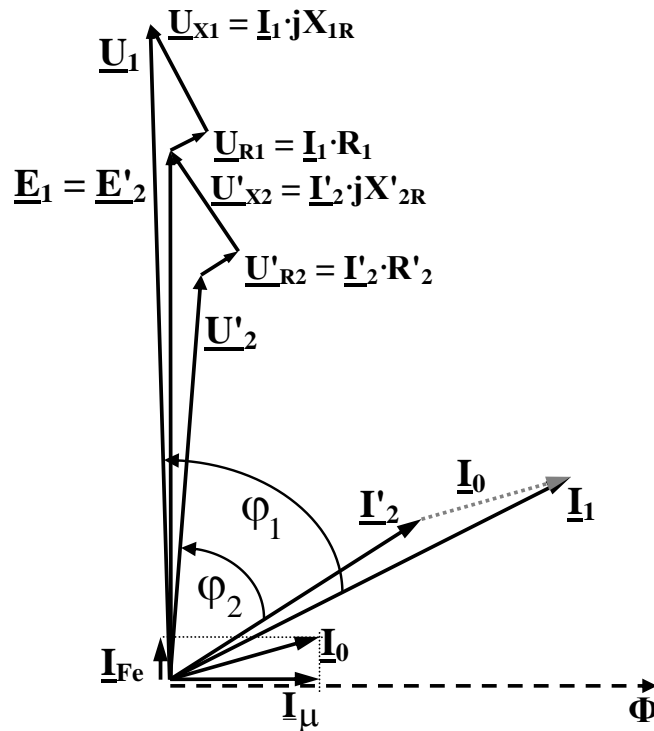


Fig. 1.7. Vector diagram of a transformer under on-load conditions



The first step in the diagram construction is to plot voltage  $\underline{U}'_2$ , i.e. the representation of element  $\underline{Z}$  voltage on the secondary side to the primary side. Knowing the element impedance, it is possible to plot current  $\underline{I}'_2$  at angle  $\varphi'_2 = \varphi_2$ . Parallel to the vector of current  $\underline{I}'_2$ , voltage drop across secondary winding resistance  $\underline{U}'_{R2}$  is plotted and perpendicular to it – voltage drops across secondary winding reactance  $\underline{U}'_{X2}$ . The end of vector  $\underline{U}'_{X2}$  determines electromotive force  $\underline{E}_1 = \underline{E}'_2$ . Parallel to the vector of EMF  $\underline{E}_1$ , the vector of core-loss current  $\underline{I}_{Fe}$  is determined, and perpendicular to it – the vector current  $\underline{I}_\mu$  (resulting from core magnetization – in order to highlight this fact, the parallel vector of main magnetic flux  $\Phi$  is marked in the diagram with a broken line). The sum of the vectors of current  $\underline{I}_{Fe}$  and  $\underline{I}_\mu$  constitutes current  $\underline{I}_0$ , which – if added to the vector of current  $\underline{I}'_2$  – makes it possible to plot the vector of current  $\underline{I}_1$  supplying the transformer. Parallel to the vector of current  $\underline{I}_1$ , voltage drop across primary winding resistance  $\underline{U}_{R1}$  is plotted and perpendicular to it – voltage drops across primary winding reactance  $\underline{U}_{X1}$ . The end of vector  $\underline{U}_{X1}$  determines the transformer required supply voltage  $\underline{U}_1$ .

### 2.3. Transformer short-circuit state

The transformer is in the short-circuit state if its primary winding is supplied with complex supply  $\underline{U}_1$  from the source, and the secondary circuit is short-circuited. In operating practice this is considered to be a failure state that has to be removed as quickly as possible. In measuring practice, the transformer short-circuit state is realized by supplying one of the windings (primary or secondary) with a voltage value that generates rated current in the supplied winding. Under the short-circuit conditions, the voltage at the terminals of the short-circuited winding is equal to zero. Current flows through the short-circuited winding but no power is given to the receiver. The total power taken from the source by a short-circuited transformer covers losses only, and is converted entirely into heat. Under short-circuit conditions, at lower supplied voltage, the power dissipated by the windings – the copper loss  $\Delta P_{Fe} = \Delta P_H + \Delta P_W$  is ignored because according to dependences (3) and (4) the loss, being dependent on the square of voltage (the value of magnetic induction is directly proportional to voltage) constitutes a fraction of percent of rated loss. For the same reason, the value of magnetizing current  $\underline{I}_\mu$  is ignored as it is of the order of a few tenths of a percent [4] of the current consumed under short-circuit conditions (i.e. of the rated current).

Considering the above, the equivalent circuit diagram and the vector diagram of a transformer under short-circuit conditions are presented in Fig. 1.8.

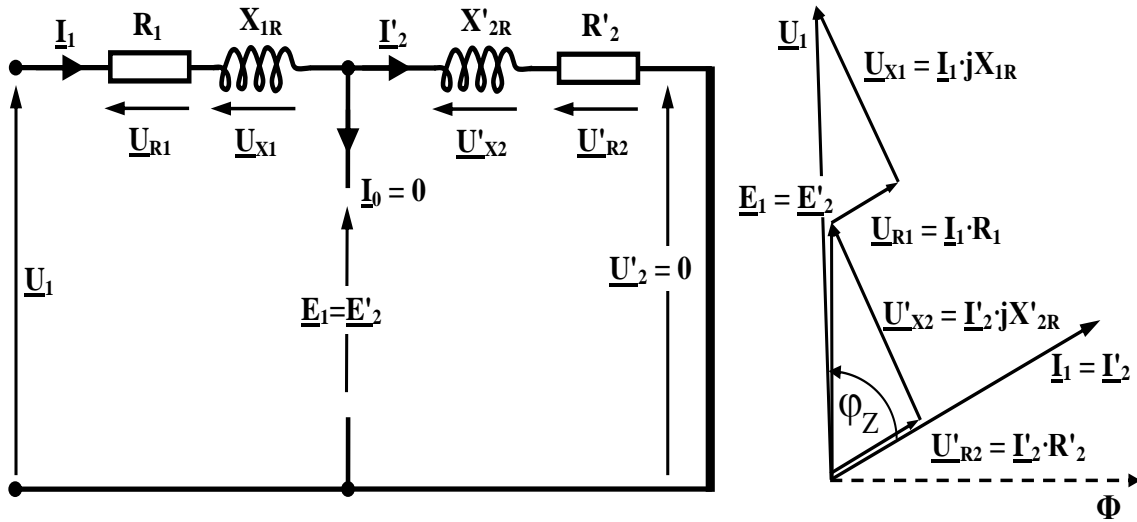


Fig. 1.8. Equivalent circuit diagram and vector diagram of a transformer under short-circuit conditions

Considering that current  $\underline{I}_1 = \underline{I}'_2$ , Kirchoff's equation for the diagram presented in Fig. 1.8 assumes the following form:

$$\underline{U}_1 = \underline{U}_{R1} + \underline{U}_{X1} + \underline{U}'_{X2} + \underline{U}'_{R2} = \underline{R}_1 \cdot \underline{I}_1 + \underline{jX}_{1R} \cdot \underline{I}_1 + \underline{jX}'_{2R} \cdot \underline{I}_1 + \underline{R}'_2 \cdot \underline{I}_1 \quad (22)$$

or

$$\underline{U}_1 = [(\underline{R}_1 + \underline{R}'_2) + \underline{j}(\underline{X}_{1R} + \underline{X}'_{2R})] \cdot \underline{I}_1 = \underline{Z}_Z \cdot \underline{I}_1 \quad (23)$$

where:

$$\underline{R}_Z = \underline{R}_1 + \underline{R}'_2 \quad - \quad \text{short-circuit resistance,}$$

$$\underline{X}_Z = \underline{X}_1 + \underline{X}'_2 \quad - \quad \text{short-circuit reactance,}$$

$$\underline{Z}_Z = \underline{R}_Z + \underline{jX}_Z \quad - \quad \text{short-circuit impedance.}$$

While one winding of the transformer is short-circuited, the other winding is supplied with voltage  $U_Z$  (short-circuit voltage) with a value causing rated current  $I_N$  to flow through the supplied winding. Measuring power  $P_Z$  and voltage  $U_Z$  and knowing the value of rated current  $I_N$ , the following may be determined:

- power dissipated by the windings (copper-loss)  $\Delta P_{Cu} = P_Z$ , (24)

- short-circuit state power factor  $\cos \varphi_Z = \frac{P_Z}{U_Z \cdot I_N}$ , (25)

- short-circuit impedance  $Z_Z = \frac{U_Z}{I_N}$ , (26)

- short-circuit resistance  $R_Z = \frac{\Delta P_{Cu}}{I_N^2} \approx \frac{P_Z}{I_N^2},$  (27)

- short-circuit reactance  $X_Z = \sqrt{Z_Z^2 - R_Z^2},$  (28)

and approximate values of the equivalent circuit diagram parameters:  $R_1, R_2, X_{1R}, X_{2R}$  calculated for the transformer turns ratio  $g_N$  assuming that  $R_1 = R'_2$  and  $X_{1R} = X'_{2R}$  [4]:

$$R_1 \approx \frac{R_Z}{2}, \quad (29)$$

$$R_2 \approx \frac{R_1}{g_N^2}, \quad (30)$$

$$X_{1R} \approx \frac{X_Z}{2}, \quad (31)$$

$$X_{2R} \approx \frac{X_{1R}}{g_N^2}. \quad (32)$$

The measurements performed under short-circuit conditions together with those made under no-load conditions make it possible to determine approximate values of parameters of all the elements of the transformer equivalent circuit diagram.

### 3. Laboratory tests and measurements

#### 3.1. Determination of measured quantities

The quantities to be measured are as follows: current intensity, active power and voltages of the primary and secondary side of the transformer under no-load conditions, as well as current intensity, active power and voltage of the transformer under short-circuit conditions. Based on the measurement data, the following are to be found: the characteristics of the no-load and short-circuit operation and the values of parameters of all the elements of the transformer equivalent circuit diagram.

#### 3.2. Determination of characteristics of the transformer no-load state

##### 3.2.1. Diagram of the test stand

The test stand is supplied from an adjustable alternating current source – autotransformer ATr. The measuring system is presented in Fig. 1.9.



## ATTENTION:

No operation such as switching on the supply or selecting or changing the measuring ranges of instruments may be performed without the class instructor's prior acceptance and supervision. **The measuring system is not separated from the supply network!**

### 3.3. Determination of the transformer short-circuit characteristics

#### 3.3.1. Diagram of the test stand

The test stand is supplied from an adjustable alternating current source – autotransformer ATr. The measuring system is presented in Fig. 2.0.

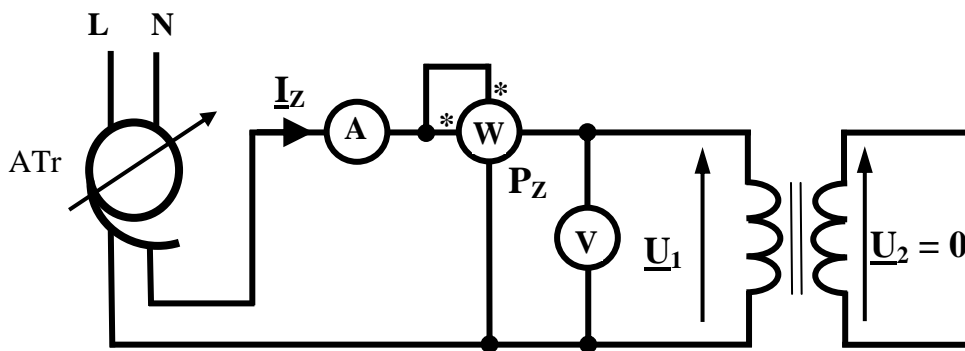


Fig. 2.0. Diagram of the measuring system for the transformer testing under short-circuit conditions

#### 3.3.2. Course of exercise

1. Arrange the measuring system according to Fig. 2.0 and report to the class instructor that the system is ready to be supplied.
2. Measure active power  $P_Z$  and intensity of current  $I_Z$  for values of voltage  $U_1$  set subsequently by means of the autotransformer (*suggested values of voltage will be specified by the class instructor – the value of **voltage at rated current** of the primary side must not be omitted*).
3. Record the results of the measurements systematically in Table 1.2.
4. After a measuring series is completed, set the transformer adjustment knob to the minimum value and switch off the supply.
5. Fill in the computational part of Table 1.2 using dependences (25) to (28) and (31) to (32).

Table 1.2

Item	Measurements			Calculations						
	$U_1$	$I_Z$	$P_Z$	$\cos \varphi_Z$	$\varphi_Z$	$R_Z$	$Z_Z$	$X_Z$	$X_{1R}$	$X_{2R}$
	V	A	W	—	—	$\Omega$	$\Omega$	$\Omega$	$\Omega$	$\Omega$
1.										
2.										
3.										
etc.										

#### 4. Elaboration on measurement results

Based on the measurement results:

- Plot the characteristics of the transformer no-load state  $I_0$ ,  $P_0$ ,  $U_2$ ,  $\cos \varphi_0$ ,  $\varphi_0$ ,  $\mathcal{G}$ ,  $I_{Fe}$ ,  $I_{\mu}$ ,  $R_{Fe}$ ,  $X_{\mu}$  in the function of supply voltage  $U_1$  (*the characteristics shown in a single chart must differ in the line colour and/or type and description*).
- Plot the characteristics of the transformer short-circuit state  $I_Z$ ,  $P_Z$ ,  $\cos \varphi_Z$ ,  $\varphi_Z$ ,  $R_Z$ ,  $Z_Z$ ,  $X_Z$  in the function of supply voltage  $U_1$  (*the characteristics shown in a single chart must differ in the line colour and/or type and description*).
- Draw a complete equivalent circuit diagram of the transformer (like for the on-load state) and specify the values of the determined parameters on it:
  - $R_1$ ,  $R_2$  – from measurements under short-circuit conditions for the rated current and, for comparison (*in parentheses*), from direct measurements (cf. 3.2.2),
  - $X_{1R}$ ,  $X_{2R}$  – from measurements under short-circuit conditions for the rated current,
  - $R_{Fe}$ ,  $X_{\mu}$  – from measurements under no-load conditions for the rated voltage.
- Determine the following values:
  - transformer ratio –  $\mathcal{G}$ ,
  - short-circuit voltage –  $u_Z$ ,
  - core loss –  $\Delta P_{Fe}$ ,
  - copper loss –  $\Delta P_{Cu}$ .

## 5. Report

The report must include:

1. The title page (*exercise name, section number, the last and first names of the students doing the exercise and the exercise date*).
2. Rated data of the transformer under analysis.
3. Diagrams of the measuring systems.
4. Tables listing the measurement results from all test stands with calculations.
5. Charts for dependences described in 4.1 and 4.2 above.
6. The transformer equivalent circuit diagram with values of the parameters specified in 4 above.
7. Remarks and conclusions (*concerning the characteristics, their deviations from theoretical characteristics, values of determined parameters of the equivalent circuit diagram, discrepancies between approximate values of the windings resistance calculated under short-circuit conditions and their measured values, etc.*).